Serie 05 - Solution

Preamble

Diffusion of minority carriers in the quasi-neutral regions

As we saw in Lecture 5, two opposite approximations may hold when calculating the distributions of minority carrier concentrations in the QNRs, depending on the dimensions of the PN junction.

1. **Short neutral sides**: if $W_p - x_p << L_n$, recombination occurs mainly at the interface with the QNR and bulk recombination can be neglected. The excess minority electrons concentration $n'(x)$ is **approximately linear with** x, in this case. The same applies to excess minority holes $p'(x)$ when $W_n - x_n << L_p$.
2. **Long neutral sides**: if $W_p - x_p >> L_n$, recombination in the bulk cannot be neglected. The excess minority electrons concentration $n'(x)$ follows an **exponential decay with** x, in this case. The same applies to excess minority holes $p'(x)$ when $W_n - x_n >> L_p$.

We remind here the formulas for the carriers diffusion lengths: $L_n = \sqrt{D_n \tau_{n0}}$, $L_p = \sqrt{D_p \tau_{p0}}$.

Given constants

$n_i(Si) = 1.5 \cdot 10^{10}\,[cm^{-3}] \quad @ \quad T = 300\,[K]$
$k = 8.62 \cdot 10^{-5}\,[eV/K]$
$q = 1.60 \cdot 10^{-19}\,[C]$
$\epsilon_0 = 8.85 \cdot 10^{-14}\,[F/cm]$
$\epsilon_{Si} = 11.7 \cdot \epsilon_0$

Exercise 01

Consider an ideal abrupt PN juntion with short neutral sides compared to the diffusion lengths of both electrons and holes: $W_n = W_p = 2\,[um]$, while we neglect the width of the depleted regions. Draw the ideal current-voltage characteristic and calculate the reverse saturation current I_S for a cross sectional area $A = 2 \cdot 10^{-4}\,\left[cm^2\right]$ and $T = 300\,[K]$. Evaluate the current through the junction with a forward bias $V_D = 0.75\,[V]$. Comment on the results.
The semiconductor parameters are: $N_A = 5 \cdot 10^{16}\,\left[cm^{-3}\right]$, $N_D = 1 \cdot 10^{16}\,\left[cm^{-3}\right]$, $D_n = 21\,\left[cm^2/s\right]$, $D_p = 10\,\left[cm^2/s\right]$.

Solution

The PN junction has short neutral sides, which means the distribution of minority carriers in the QNRs is linear. We use the corresponding formula for the saturation current:

$$I_S \approx Aqn_i^2 \left(\frac{D_n}{W_p N_A} + \frac{D_p}{W_n N_D} \right) = 5.11 \cdot 10^{-14} \, [A] \tag{1}$$

where we neglected the widths of the depletion regions x_p and x_n.
We can then easily calculate the current at the setpoint $V_D = 0.75 \, [V]$:

$$I = I_S \left[exp \left(\frac{V_D}{V_T} \right) - 1 \right] \approx 0.172 \, [A] \tag{2}$$

The reverse saturation current is the current flowing through the junction under reverse bias. It is in the order of the fA in a typical device, which is negligible. In contrast, there is a huge increase of the current in forward bias (that depends exponentially on the thermal voltage $V_T = \frac{kT}{q}$). We call this property current rectification of the PN junction.

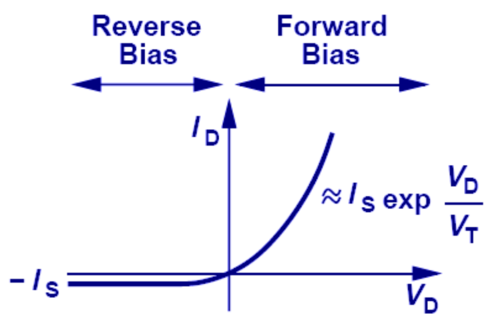

Figure 1: Static characteristic of an ideal PN junction.

Exercise 02

Design the doping concentrations N_A and N_D of a PN diode such that its diffusion current densities for electrons and holes are respectively $J_n = 20 \left[A/cm^2\right]$ and $J_p = 5 \left[A/cm^2\right]$ at an applied forward bias $V_D = 0.65 \, [V]$ ($T = 300 \, [K]$). The diode has fixed dimensions $W_n = W_p = 300 \, [um]$ and the semiconductor parameters are: $\tau_{n0} = \tau_{p0} = 5 \cdot 10^{-7} \, [s]$, $D_n = 25 \left[cm^2/s\right]$, $D_p = 10 \left[cm^2/s\right]$.

Solution

The first point is to determine whether we are in the short or long neutral sides conditions. We calculate the diffusion lengths for both carriers:

$$L_n = \sqrt{D_n \tau_{n0}} \approx 35.4\,[um] \tag{3}$$

$$L_p = \sqrt{D_p \tau_{p0}} \approx 22.4\,[um] \tag{4}$$

which are one order of magnitude smaller than the p- and n-side lengths.
We recall that the long neutral sides conditions are $W_p - x_p >> L_n$ and $W_n - x_n >> L_p$, which also depend on the width of the depleted regions. Since we cannot calculate x_p and x_n for now, we assume the two conditions to hold and then verify this assumption at the end, once we defined the doping concentrations.
The current densities for long neutral sides are:

$$J_n = q \frac{n_i^2}{N_A} \frac{D_n}{L_n} \left[exp\left(\frac{V_D}{V_T}\right) - 1 \right] \tag{5}$$

$$J_p = q \frac{n_i^2}{N_D} \frac{D_p}{L_p} \left[exp\left(\frac{V_D}{V_T}\right) - 1 \right] \tag{6}$$

Substituting all quantities, we obtain $N_A = 9.15 \cdot 10^{14}\,\left[cm^{-3}\right]$ and $N_D = 2.31 \cdot 10^{15}\,\left[cm^{-3}\right]$.
We now calculate the width of the depletion region at $V_D = 0.65\,[V]$ to make sure the long neutral sides conditions still hold. We just need to calculate the built-in potential first:

$$\phi_b = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right) = 597\,[mV] \tag{7}$$

$$x_d\,(V_D) = x_n\,(V_D) + x_p\,(V_D) = \sqrt{\frac{2\epsilon_{Si}\,(N_A + N_D)}{q N_A N_D}\,(\phi_b - V_D)} \tag{8}$$

In this case, the applied bias even exceeds the built-in potential, meaning the depletion region is really really thin. The long neutral sides conditions are verified.
For completeness, we report here the width of the depletion region without an external bias ($V_D = 0\,[V]$):

$$x_{d0} = x_{n0} + x_{p0} = \sqrt{\frac{2\epsilon_{Si}\phi_b\,(N_A + N_D)}{q N_A N_D}} \approx 1.41\,[um] \tag{9}$$

Exercise 03

Consider a Si PN junction initially biased at $V_D = 0.6\,[V]$ at $T = 300\,[K]$. The temperature later increases to $T = 310\,[K]$. Calculate the change in the

forward-bias voltage V_D required to maintain a constant current through the junction. We assume dopants are completely ionized at $T = 300\,[K]$.
Reminder: $n_i^2 \propto exp\left(\frac{-E_g}{kT}\right)$ with $E_g = 1.12\,[eV]$ for silicon.

Solution

We can write the diode current as follows, highlighting its temperature dependencies (check formulas 1, 2):

$$I \propto n_i^2(T)\left[exp\left(\frac{qV_D}{kT}\right) - 1\right] \approx exp\left(\frac{-E_g}{kT}\right) exp\left(\frac{qV_D}{kT}\right) \tag{10}$$

The approximation neglects the reverse-bias current. This is quite accurate as the junction is in forward-bias with $V_D = 0.6\,[V]$.
If the temperature changes, we may take the ratio of the diode currents at two temperatures. This ratio is:

$$\frac{I_2}{I_1} = \frac{exp\left(\frac{-E_g}{kT_2}\right) exp\left(\frac{qV_{D2}}{kT_2}\right)}{exp\left(\frac{-E_g}{kT_1}\right) exp\left(\frac{qV_{D1}}{kT_1}\right)} \tag{11}$$

If the current is to be held constant, then $I_1 = I_2$ and we must have

$$\frac{qV_{D1} - E_g}{kT_1} = \frac{qV_{D2} - E_g}{kT_2} \tag{12}$$

Let $T_1 = 300\,[K]$, $T_2 = 310\,[K]$, $E_g = 1.12\,[eV]$ and $V_{D1} = 0.6\,[V]$, we obtain $V_{D2} \approx 0.583\,[V]$

Exercise 04

Calculate the small-signal admittance, small-signal resistance and the diffusion capacitance of a PN junction diode. Assume $N_A >> N_D$, so that $p_{n0} >> n_{p0}$ and therefore $I_{p0} >> I_{n0}$. Let $T = 300\,[K]$, $\tau_{p0} = 1 \cdot 10^{-7}\,[s]$ and $I_{p0} = 1\,[mA]$. The transit time can be approximated to $\tau_T \approx \tau_{p0}/2$.

Solution

The small signal admittance is:

$$g_d = \frac{qI}{kT} \approx \frac{qI_{p0}}{kT} = 38.5\,[mS] \tag{13}$$

hence, the small-signal resistance is:

$$r_d = \frac{1}{g_d} = 26\,[\Omega] \tag{14}$$

Finally, the diffusion capacitance is given by:

$$C_d \approx \frac{qI_{p0}\tau_{p0}}{2kT} = 1.92\,[nF] \tag{15}$$